REVIEW OF THE PAPER "Bounds for the Convex Combination of the First Seiffert and Logarithmic Means in Terms of Generalized Heronian Mean
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The paper concerns interesting properties of classical means. It present results obtained by using well-known methods. The proof of the main theorem is rather technical, but the result is interesting and correct. Therefore the result is possible to publish in the journal. But, the paper should be corrected.

My remarks:

1. The bibliography is very poor. We can find a lot papers related to the topics investigated by the authors (e.g. listed below). The obtained result generalize results obtained in some of these papers. The authors should include suitable chosen articles to the references.

2. Also, the introduction is poor. It should include a motivation of the investigation, in particular, its connections with the other results in the field (see for example introductions in the papers below).

3. The formulation of Theorem 2.1 is unclear and incorrect. The essence of Theorem 2.1 should be the following

   - Let $\alpha \in [0,1]$. Then
   $$H_p(a,b) < \alpha P(a,b) + (1-\alpha)L(a,b), \quad a,b>0,\ a=b, \tag{1}$$
   if and only if $p < p(\alpha) := \frac{\pi}{\alpha} - 2$.
   - If
   $$\alpha P(a,b) + (1-\alpha)L(a,b) < H_p(a,b), \quad a,b>0,\ a=b, \tag{2}$$
   then $q < 4 - 3\alpha$. Moreover, if $q < \frac{4}{1+3\alpha}$, then (2) holds.

4. The list of additional remarks and mistakes:

   - the optimal problem and the optimal functions should be defined in Introduction
   - the optimal inequalities (1) and (2) are not clearly written
   - the lower limit should be inserted in (5)
   - $\frac{1}{b}$ should be inserted in (7) after the last inequality
   - what does it means the numerator of $h^0(t)$ in (12)?
   - "$(4+3\alpha)$" should be replaced by "$(1+3\alpha)$" on page 3, line20
   - I propose to replace the functions "ss, pp, rr, etc. " by s_e, p_e, r_e, etc."

5. The manuscript should be checked for English by a language editor.

Taking the above into consideration I cannot recommend the publication of the manuscript in the present form. But, in my opinion it is possible to accept the paper after having make the ma jor revision. Then, further review is required after correction of the manuscript.

The Reviewer, January 2012

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I agree with reviewer comments. I made all corrections.

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